

These unique tables consist of 5D values of the Jacobian elliptic functions $\operatorname{sn}(w, k)$, $\operatorname{cn}(w, k)$, and $\operatorname{dn}(w, k)$, where $w = u + iv$, as functions of Jacobi's nome q , which equals $\exp(-\pi K'/K)$, where K and K' are the quarter-periods (the complete elliptic integrals of the first kind of modulus k and of complementary modulus k' , respectively).

The range of parameters in the table is: $q = 0.005(0.005)0.480$, $u/K = 0(0.1)1$, and $v/K' = 0(0.1)1$. For larger values of q the authors give in the introduction approximations to the elliptic functions by circular and hyperbolic functions.

These tables were computed on an IBM 7094 system by the method of modulus reduction based on Gauss's transformation. Essentially the same subroutine was used here as in the calculation of an earlier table [1] of Jacobian elliptic functions by the same authors.

Reference should also be made to a manuscript table [2] of elliptic functions for complex arguments by these authors, which, however, has $\sin^{-1} k$ for an argument in place of q .

J. W. W.

1. HENRY E. FETTIS & JAMES C. CASLIN, *Ten Place Tables of the Jacobian Elliptic Functions*, Report ARL 65-180, Part 1, Aerospace Research Laboratories, Wright-Patterson Air Force Base, Ohio, September 1965. (See *Math. Comp.*, v. 21, 1967, pp. 264-265, RMT 25.)

2. HENRY E. FETTIS & JAMES C. CASLIN, *Jacobian Elliptic Functions for Complex Arguments*, ms. deposited in UMT file. (See *Math. Comp.*, v. 21, 1967, p. 508, RMT 65.)

19 [7].—V. M. BELIAKOV, R. I. KRAVTŠOVA & M. G. RAPPAPORT, *Tablitsy elliptičeskiĭkh integralov*, Tom II (*Tables of Elliptic Integrals*, Vol. II), Izdatel'stvo Akademii Nauk SSSR, Moscow, 1963, xii + 783 pp., 27 cm. Price 6 rubles, 18 kopecks.

This set of tables is a continuation of the systematic tabulation (without differences) of the elliptic integral of the third kind $\prod(n, k^2, \phi)$ to 7S which was carried out for negative values of the parameter n in the first volume [1]. In this second volume, n assumes nonnegative values; specifically, $n = 0(0.1)1, 1.2, 1.5(0.5)5(1)10, 12, 15(5)40(10)100$, while the ranges of k and ϕ are the same as in the first volume; namely, $k^2 = 0(0.01)1$ and $\phi = 0^\circ(1^\circ)90^\circ$.

To this main table of 728 pages there are appended six supplementary tables. The first four of these are of $T_n^\epsilon(k^2, \phi) = \int_\epsilon^\phi \sin^{-2n} \alpha (1 - k^2 \sin^2 \alpha)^{-1/2} d\alpha$ for $k^2 = 0(0.01)1$, $35^\circ \leq \phi \leq 90^\circ$, $\epsilon = 35^\circ$, and $n = 1, 2, 3, 4$, respectively. The precision of these four tables is 7S, 5S, 3S and 2S, respectively. They are provided to facilitate the evaluation of

$$\begin{aligned} \prod_\epsilon(n, k^2, \phi) &= \int_\epsilon^\phi (1 + n \sin^2 \alpha)^{-1} (1 - k^2 \sin^2 \alpha)^{-1/2} \\ &= \sum_{m=1}^{\infty} (-1)^{m+1} T_m^\epsilon(k^2, \phi) n^{-m}, \end{aligned}$$

where $\epsilon = 35^\circ$ and $n > 100$.

The calculation of $\prod(n, k^2, \phi)$ when $0^\circ < \phi \leq 35^\circ$ and $n > 100$ can be effected by two series (according as $k^2 \leq 0.7$ or $k^2 \geq 0.7$) that involve, respectively, $A_m(\phi) = \int_0^\phi \sin^{2m} \alpha d\alpha$ and $R_m(\phi) = \int_0^\phi \tan^{2m} \alpha \sec \alpha d\alpha$. These functions are tabulated to 8D or 8S in the first volume for $m = 1(1)10$ and $m = 0(1)8$, respectively.

A detailed discussion of these series appears in the Introduction to the present tables (pp. v-vi).

The remaining two auxiliary tables consist, respectively, of $\sin \phi$, $\cos \phi$, $\phi = 0^\circ(1^\circ)90^\circ$, 10D; $\tan \phi$, $\cot \phi$, $\phi = 0^\circ(1^\circ)90^\circ$, 11S and 12S; and of $R_0 = \ln \tan(\phi/2 + \pi/4)$, $\phi = 0^\circ(1^\circ)89^\circ$, 9D.

Appended to the Introduction is a bibliography of 11 items, which, however, does not include a reference to the tables of Paxton & Rollin [2]. Moreover, since the publication of these Russian tables, Fettis & Caslin have calculated 10D tables of elliptic integrals of all three kinds [3]; however, therein the tabulation of the integral of the third kind is over a broader mesh in k and ϕ and is restricted to $-1 \leq n \leq 1$, in contrast to the tables under review, which are by far the most elaborate of their kind calculated to date.

J. W. W.

1. V. M. BELĀKOV, R. I. KRAVŤSOVA & M. G. RAPPAPORT, *Tablitsy elliptičeskikh integralov*, Tom I, Izdatel'stvo Akademii Nauk SSSR, Moscow, 1962. (See *Math. Comp.*, v. 18, 1964, pp. 676-677, RMT 93; v. 19, 1965, p. 694, RMT 127.)

2. F. A. PAXTON & J. E. ROLLIN, *Tables of the Incomplete Elliptic Integrals of the First and Third Kind*, Curtiss-Wright Corporation, Research Division, Quehanna, Pa., June 1959. (See *Math. Comp.*, v. 14, 1960, pp. 209-210, RMT 33.)

3. HENRY E. FETTIS & JAMES C. CASLIN, *Tables of Elliptic Integrals of the First, Second, and Third Kind*, Applied Mathematics Research Laboratory Report ARL 64-232, Aerospace Research Laboratories, Wright-Patterson Air Force Base, Ohio, December 1964. (See *Math. Comp.*, v. 19, 1965, p. 509, RMT 81. For errors, see *Math. Comp.*, v. 20, 1966, pp. 639-640, MTE 398.)

20 [9].—M. LAL, *Decimal Expansion of Mersenne Primes*, ms. of 19 pp., dated June 20, 1967, deposited in the UMT file.

This manuscript contains the exact values in the decimal system of those thirteen known Mersenne primes M_p for which $p > 100$, calculated on an IBM 1620 at Dalhousie University.

A table is included which gives for each of these primes the frequency distribution of the digits, with the corresponding χ^2 value, and the total number of digits. No significant departure from a random distribution of the digits can be inferred from this statistical analysis.

The author notes in his introductory remarks that Sierpiński [1] gives the number of digits in M_{1279} and M_{11213} incorrectly as 376 and 3381, respectively, instead of 386 and 3376. In addition, the present author has observed that Hardy & Wright [2] give the latter number incorrectly as 3375.

For supplementary information the author refers the reader to papers by Gillies [3] and Uhler [4], [5].

J. W. W.

1. W. SIERPIŃSKI, *Elementary Theory of Numbers*, Państwowe Wydawnictwo Naukowe (Polish Scientific Publishers), Warsaw, and Hafner Publishing Co., New York, 1964, p. 341.

2. G. H. HARDY & E. M. WRIGHT, *An Introduction to the Theory of Numbers*, 4th ed., 1960, reprinted 1965, p. 16.

3. D. B. GILLIES, "Three new Mersenne primes and a statistical theory," *Math. Comp.*, v. 18, 1964, pp. 93-95.

4. H. S. UHLER, "A brief history of the investigation on Mersenne numbers and the latest immense primes," *Scripta Math.*, v. 18, 1952, pp. 122-131.

5. H. S. UHLER, "On the 16th and 17th perfect numbers," *Scripta Math.*, v. 19, 1953, pp. 128-131.